

I. (Definitions) Match the following items with correct definitions.

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|----|-----------------------|---|
| a. | $T(n) = O(f(n))$ | 1. There exist $c, n_0 > 0$ such that
for all $n \geq n_0$, $T(n) \leq cf(n)$ |
| b. | $T(n) = \Theta(f(n))$ | 2. There exist $c, n_0 > 0$ such that
for all $n \geq n_0$, $T(n) \geq cf(n)$ |
| c. | $T(n) = \Omega(f(n))$ | 3. There exist $c_1, c_2, n_0 > 0$ such that
for all $n \geq n_0$, $c_1f(n) \leq T(n) \leq c_2f(n)$ |

2. Let $f(n) = n^2 + 2n$ and $g(n) = n^2$. Answer to the following questions. Give proofs or counterexamples supporting your answer.

i. $f(n) = O(g(n))$

ii. $f(n) = \Omega(g(n))$

iii. $f(n) = \Theta(g(n))$

3. Let $f(n) = n^2$ and $g(n) = n$. Answer to the following questions. Give proofs or counterexamples supporting your answer.

i. $f(n) = O(g(n))$

ii. $f(n) = \Omega(g(n))$

iii. $f(n) = \Theta(g(n))$

4. Solve the following recurrences.

i. $T(n) = T(n-1) + n$

ii. $T(n) = 2T(n/2) + n$

iii. $T(n) = 3T(n/4) + n^2$

iv. $T(n) = 5T(n/2) + n^2$